

H-Dibaryon from Lattice QCD with Improved Anisotropic Actions

Zhi-Huan Luo^{*}Mushtaq Loan[†]and Xiang-Qian Luo[‡]

March 24, 2008

Abstract

The six quark state(uuddss) called H dibaryon($J^P = 0^+, S = -2$) has been calculated to study its existence and stability. The simulations are performed in quenched QCD on $8^3 \times 24$ and $16^3 \times 48$ anisotropic lattices with Symanzik improved gauge action and Clover fermion action. The gauge coupling is $\beta = 2.0$ and aspect ratio $\xi = a_s/a_t = 3.0$. Preliminary results indicate that mass of H dibaryon is 2134(100)MeV on $8^3 \times 24$ lattice and 2167(59)MeV on $16^3 \times 48$ respectively. It seems that the radius of H dibaryon is very large and the finite size effect is very obvious.

1 Introduction

In 1976, Jaffe pointed out that the quark bag model predicted the existence of H dibaryon, which is a compound state of 6 quarks(uuddss)[1]. It is the lowest bound state in dibaryon sector and will be a spin 0 strangeness -2 SU(3) flavor singlet. Jaffe's original bag model suggested that $m_H = 2150\text{MeV}$, 81MeV below the 2231MeV $\Lambda\Lambda$ threshold.

Since Jaffe's prediction, people tried to find out the H-Dibaryon state both by experiment and theoretical calculation. On the experimental side, until now, there are no enough evidences on the existence of the H-Dibaryon. On the theoretical side, many theoretical calculations have been made to predict the mass of H dibaryon. One of the most efficient ways to study this state is from the first principle of QCD, i.e Lattice QCD. Many simulation results suggested that H dibaryon is a bound state, but some gave contrary conclusions [2, 3, 4, 5, 6].

We perform the numerical simulations with refined methods, which includes improved gauge and fermion action, smearing techniques and tadpole improvement. To reduce computer cost and determine large masses more accurately, we do the simulations on anisotropic lattice for both gauge action and fermion action. This is our advantage comparing with what the forthgoer have done.

^{*}email: lozit@scau.edu.cn

[†]email: stsmushe@zsu.edu.cn

[‡]email: stslxq@zsu.edu.cn

2 H-Dibaryon Simulation Details

2.1 Actions

We generate the configurations using improved,anisotropic action[7]:

$$S_g = \beta \left\{ \frac{5}{3} \frac{\Omega_{sp}}{\xi u_s^4} + \frac{4}{3} \frac{\xi \Omega_{tp}}{u_s^2 u_t^2} - \frac{1}{12} \frac{\Omega_{sr}}{\xi u_s^6} - \frac{1}{12} \frac{\xi \Omega_{str}}{u_s^4 u_t^2} \right\}, \quad (1)$$

where $\beta = 6/g^2$, g is the QCD coupling, u_s and u_t are the mean-link renormalization parameters, ξ is the aspect ratio ($\xi = a_s/a_t$ at the tree level in perturbation theory),and

$$\begin{aligned} \Omega_{sp} &= \sum_x \sum_{i>j} \frac{1}{3} \text{ReTr}[1 - U_i(x)U_j(x+i)U_i^\dagger(x+j)U_j^\dagger(x)], \\ \Omega_{tp} &= \sum_x \sum_i \frac{1}{3} \text{ReTr}[1 - U_i(x)U_t(x+i)U_i^\dagger(x+t)U_t^\dagger(x)], \\ \Omega_{sr} &= \sum_x \sum_{i \neq j} \frac{1}{3} \text{ReTr}[1 - U_i(x)U_i(x+i)U_j(x+2i)U_i^\dagger(x+j+i)U_i^\dagger(x+j)U_j^\dagger(x)], \\ \Omega_{str} &= \sum_x \sum_i \frac{1}{3} \text{ReTr}[1 - U_i(x)U_i(x+i)U_t(x+2i)U_i^\dagger(x+t+i)U_i^\dagger(x+t)U_t^\dagger(x)], \end{aligned}$$

where x labels the sites of the lattice, i,j are spatial indices, and $U_\mu(x)$ is the parallel transport matrix in the gauge field from site x to $x + \mu$.

For the quark action, we employ the space-time asymmetric clover quark action on anisotropic lattice[8][9][10]:

$$\begin{aligned} S_f &= \sum_x \bar{\Psi}_x \Psi_x \\ &- K_s \sum_x \sum_i [\bar{\Psi}_x(r_s - \gamma_i)U_{i,x}\Psi_{x+i} + \bar{\Psi}_x(r_s + \gamma_i)U_{i,x-i}^\dagger\Psi_{x-i}] \\ &- K_t \sum_x [\bar{\Psi}_x(1 - \gamma_t)U_{t,x}\Psi_{x+t} + \bar{\Psi}_x(1 + \gamma_t)U_{t,x-t}^\dagger\Psi_{x-t}] \\ &+ iK_s c_s \sum_{x,i < j} \bar{\Psi}_x \sigma_{i,j} F_{ij}(x) \Psi_x + iK_s c_t \sum_{x,i} \bar{\Psi}_x \sigma_{ti} F_{ti}(x) \Psi_x, \end{aligned} \quad (2)$$

where $K_{s,t}$ and $c_{s,t}$ are the spatial and temporal hopping parameters and the clover coefficients, respectively. The hopping parameters $K_{s,t}$ are related to the bare quark mass $m_0 = a_t m_{q0}$ through

$$a_t m_{q0} \equiv 1/(2K_t) - 3r_s/\zeta - 1, \zeta = K_t/K_s. \quad (3)$$

We perform the simulations using tree-level improved Symanzik action and Clover fermion action, with gauge coupling $\beta = 2.0$ and the aspect ratio $\xi = 3.0$. The lattice sizes are $8^3 \times 24$ and $16^3 \times 48$.

2.2 Operator and Correlation Function of H Dibaryon

The operator of H-Dibaryon[11]:

$$O_H(x) = 3(udsuds) - 3(ussudd) - 3(dssduu), \quad (4)$$

$$(abcdef) = \epsilon_{abc}\epsilon_{def}(C\gamma_5)_{\alpha\beta}(C\gamma_5)_{\gamma\delta}(C\gamma_5)_{\epsilon\phi}a_\alpha^a(x)b_\beta^b(x)c_\epsilon^c(x)d_\gamma^d(x)e_\delta^e(x)f_\phi^f(x), \quad (5)$$

where a, b, c, d, e, f are color indices and $\alpha, \beta, \gamma, \delta, \epsilon, \phi$ are spinor indices.

And the corresponding correlation function of H Dibaryon can be written as $G_H(\vec{x}, \tau) = \langle O_H(\vec{x}, \tau) O_H^\dagger(0) \rangle$, which involves terms of the structure[12]:

$$(U_{11}U_{22} - U_{12}U_{21})(D_{11}D_{22} - D_{12}D_{21})(S_{11}S_{22} - S_{12}S_{21}). \quad (6)$$

To decide whether the H Dibaryon is stable or not, usually we can compare the mass of $\Lambda\Lambda$ with H-Dibaryon's. The $\Lambda\Lambda$ operator[12]:

$$O_\Lambda(x) = \epsilon_{abc}(C\gamma_5)_{\beta\gamma}[u_\alpha^a(x)d_\beta^b(x)s_\gamma^c(x) + d_\alpha^a(x)s_\beta^b(x)u_\gamma^c(x) - 2s_\alpha^a(x)u_\beta^b(x)d_\gamma^c(x)]. \quad (7)$$

2.3 Smearing Techniques

To reduce the excited-state contamination, we use the smearing techniques which can provide a better overlap with the ground state[13]. For the quark fields we use:

$$\begin{aligned} \psi'(x, R) &= \sum_{\mu \in V_z} (U^\dagger(x - \hat{\mu}) \dots U^\dagger(x - R\hat{\mu})\psi(x - R\hat{\mu}) \\ &\quad + U(x) \dots U(x + (R-1)\hat{\mu})\psi(x + R\hat{\mu})). \end{aligned} \quad (8)$$

A more large plateau in the region with small errors is obtained with smearing.

3 Simulation Results

Our results are shown in Tables 1, 2, 3 and 4. The k_{H_t} is the temporal heavy kappa, which corresponds to s quark; the k_{L_t} is temporal light kappa, which corresponds to u and d quarks.

The H-Dibaryon remain lighter than two Λ at all combinations of the hopping parameters, as shown in tables.

To obtain the physical masses of H and Λ , one has to extrapolate or interpolate the k_{L_t} and k_{H_t} to physical hopping parameters. Since $(m_\pi a)^2$ is linearly related to $1/k$, we can determine the critical hopping parameter k_c at which $(m_\pi a)^2$ vanishes. We take physical k_{ud} as k_c because they are very close. The physical k_s can be determined from the ratio of a strangeness carrying particle to a non-strange one, here we obtain the k_s by the mass ratio of lambda and

Table 1: ma_t of the Λ ($8^3 \times 24, \beta = 2.0, \xi = 3.0$)

k_{H_t}/k_{L_t}	0.23810	0.23923	0.24039	0.24155	0.24272
0.23256	1.3772(65)	1.3496(67)	1.3202(69)	1.2885(71)	1.2542(73)
0.23365	1.3665(66)	1.3388(68)	1.3093(70)	1.2777(72)	1.2433(74)
0.23474	1.3553(66)	1.3276(68)	1.2981(70)	1.2664(72)	1.2319(75)
0.23586	1.3438(67)	1.3160(69)	1.2864(71)	1.2546(73)	1.2201(75)
0.23697	1.3317(68)	1.3039(69)	1.2742(71)	1.2424(74)	1.2078(76)
0.23810	1.3189(69)	1.2910(71)	1.2613(73)	1.2294(75)	1.1947(77)

Table 2: ma_t of the Λ ($16^3 \times 48, \beta = 2.0, \xi = 3.0$)

k_{H_t}/k_{L_t}	0.23810	0.23923	0.24039	0.24155	0.24272
0.23256	1.4161(32)	1.3881(33)	1.3583(34)	1.3262(34)	1.2913(35)
0.23365	1.4052(32)	1.3772(33)	1.3473(34)	1.3152(34)	1.2802(35)
0.23474	1.3940(33)	1.3659(33)	1.3359(34)	1.3037(35)	1.2688(36)
0.23585	1.3822(33)	1.3541(34)	1.3241(34)	1.2918(35)	1.2568(36)
0.23697	1.3699(33)	1.3418(34)	1.3117(34)	1.2794(35)	1.2443(36)
0.23810	1.3568(32)	1.3286(33)	1.2985(33)	1.2661(34)	1.2309(35)

nucleon. Our calculations suggest that $k_c = 0.25256(41)$ and $k_s = 0.2413(40)$ on $8^3 \times 24$ lattice and $k_c = 0.25323(18)$ and $k_s = 0.2422(21)$ on $16^3 \times 48$ lattice.

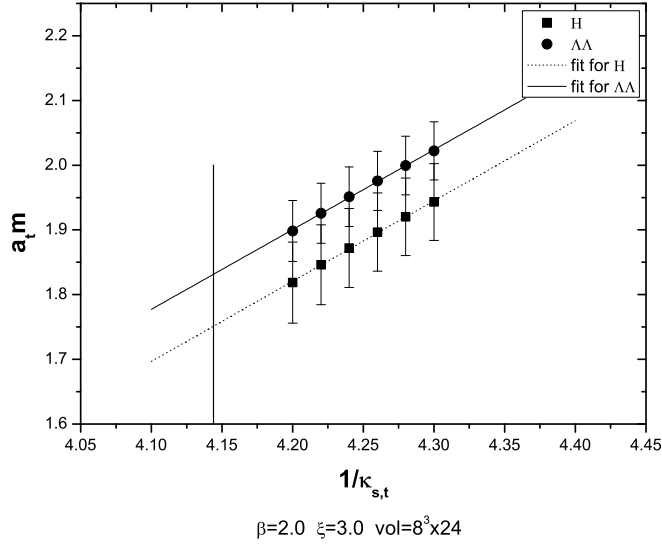
In fig.1, we performed a linear fit to extrapolate the m_H and m_Λ to the physical k_s , and obtained the H's mass $m_H = 2134(100)\text{Mev}$, which is lower than two Λ 's. The difference in mass is $m_H - 2m_\Lambda = -97(100)\text{Mev}$. That means the H-Dibaryon tends to be a bound state but actually we can't make this conclusion because the error is larger than the mass difference on $8^3 \times 24$ lattice. In fig.2, on larger lattice($16^3 \times 48$), $m_H = 2167(59)\text{Mev}$ and $m_H - 2m_\Lambda = -64(59)\text{Mev}$, indicates that the energy of H dibaryon tends to below the $\Lambda\Lambda$ threshold.

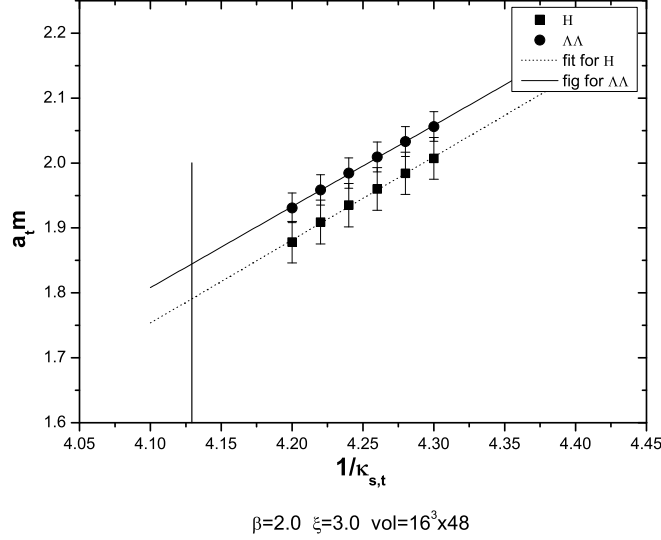
Table 3: ma_t of the H ($8^3 \times 24, \beta = 2.0, \xi = 3.0$)

k_{H_t}/k_{L_t}	0.23810	0.23923	0.24039	0.24155	0.24272
0.23256	2.656(17)	2.603(18)	2.545(18)	2.484(19)	2.417(19)
0.23365	2.635(18)	2.581(18)	2.523(18)	2.462(19)	2.395(20)
0.23474	2.613(18)	2.558(18)	2.501(19)	2.439(19)	2.372(20)
0.23586	2.589(18)	2.535(18)	2.477(19)	2.415(19)	2.348(20)
0.23697	2.565(18)	2.510(19)	2.452(19)	2.390(20)	2.323(20)
0.23810	2.539(18)	2.484(19)	2.426(19)	2.364(20)	2.297(20)

Table 4: ma_t of the H ($16^3 \times 48, \beta = 2.0, \xi = 3.0$)

k_{H_t}/k_{L_t}	0.23810	0.23923	0.24039	0.24155	0.24272
0.23256	2.7641(87)	2.7093(90)	2.6508(93)	2.5882(97)	2.5203(102)
0.23365	2.7422(88)	2.6872(91)	2.6287(94)	2.5660(98)	2.4980(103)
0.23474	2.7194(89)	2.6643(92)	2.6057(95)	2.5429(99)	2.47489(105)
0.23585	2.6956(90)	2.6405(93)	2.5818(97)	2.5188(101)	2.4507(106)
0.23697	2.6708(91)	2.6155(94)	2.5567(98)	2.4937(102)	2.4254(108)
0.23810	2.6407(86)	2.5854(88)	2.5265(92)	2.4634(96)	2.3951(101)





4 Conclusion and Future Plans

We have presented the results of the lattice investigation on the H dibaryon state employing anisotropic improved gauge and anisotropic clover fermion actions. The advantage of using anisotropic QCD actions is to get a better signal so as to obtain a large plateau on small lattice size which can save us much more computer cost. In the mean time, the simulation results are more accurately.

Our results indicate that, both on the $8^3 \times 24$ and $16^3 \times 24$ anisotropic lattice, the masses of H dibaryon are less than that of two Λ s. It seems that the H dibaryon do exist as a bound state.

The masses of H Dibaryon on two different lattices are not so close and we believe that the finite size effect of H dibaryon should be taken into account. We plan to calculate the H dibaryon on larger lattice size to further study the finite size effect. We also intend to calculate other six quarks states, such as possible proton-antiproton, deuteron, and so on. It should be very interesting.

5 Acknowledgement

The code is based on MILC's Code. The authors are grateful to Carleton DeTar and Ines Wetzorke for fruitful discussions, and to ICMSEC(The Institute of Computational Mathematics and Scientific/Engineering Computing of Chinese Academy of Sciences) for providing the LSSC2 cluster on which some of our simulations were performed.

References

- [1] R. L. Jaffe, Phys. Rev. Lett. 38(1977) 195.
- [2] P. B. Mackenzie, H. B. Thacker, Phys. Rev. Lett. 55(1985) 2539.
- [3] Y. Iwasaki, T. Yoshie, and Y. Tsuboi, Phys. Rev. Lett. 60(1988) 1371
- [4] J. W. Negele, A. Pochinsky and B. Scarlet, Nucl. Phys. B (Proc. Suppl.) 73(1999) 255 [arXiv:hep-lat/9809077].
- [5] I. Wetzorke, F. Karsch and E. Laermann, Nucl. Phys. Proc. Suppl. 83(2000) 218 [arXiv:hep-lat/9909037].
- [6] I. Wetzorke and F. Karsch, Nucl. Phys. B (Proc. Suppl.) 119(2003) 278 [arXiv:hep-lat/0208029].
- [7] C. J. Morningstar and M. Peardon, Phys. Rev. D 56(1997) 4043 [arXiv:hep-lat/9704011].
- [8] T. R. Klassen, Nucl. Phys. B(Proc. Suppl.) 73(1999) 918 [arXiv:hep-lat/9809174].
- [9] M. Okamoto et al., Phys. Rev. D65:094508,2002 [arXiv:hep-lat/0112020].
- [10] Z. H. Mei and X. Q. Luo, Int.J.Mod.Phys.A18:5713,2003 [arXiv:hep-lat/0206012].
- [11] E. Golowich and T. Sotirelis, Phys. Rev. D46(1992) 354
- [12] Ines Wetzorke, P.h.D thesis.
- [13] P. Lacock et al., Phys. Rev. D51(1995) 6403